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Revenue Management Problem in the Aviation Industry with Optimal Seat Allocation Model

Mio Imai *, Tetsuya Sato *, Takayuki Shiina *

Abstract

This study presents an optimization model that uses stochastic programming to optimally allocate seats and maximize the profit of the airline, considering of overbooking. Airline seat inventory control involves selling the right seats to the right people at the right time. If an airline sells tickets on a first-come, first-serve basis, it is likely to be occupied by leisure travelers and late bookers. Therefore, business travelers willing to pay a higher fare will subsequently find no seats left, and revenue from such sales will be lost. While there are various needs that depend on the type of passenger, this study proposes an optimization model that uses stochastic programming as a method of maximizing the profit of the airline company by allocating seats appropriately and employing the concept of overbooking.

Keywords: revenue management, stochastic programming, overbooking

1 Introduction

Since the late 1970s, following the deregulation in the United States, revenue management of airlines began, and each company was permitted to set its own fares. Tickets for the same destination are classified into various classes and subsequently sold. Airlines adjust the price and number of seats in each class to reduce the number of vacant seats at takeoff. When selling tickets on a first-come, first-serve basis, there is a tendency to sell tickets from the low-priced class to ensure that all seats are booked; such tickets are predominantly bought by discount-seeking passengers (mostly leisure travelers) who prefer to reserve low-priced seats at early stages. The companies thus struggle to sell higher-priced tickets that they could have sold to business passengers. To prevent such losses, it is necessary to allocate seats appropriately by employing the concept of overbooking, which refers to the acceptance of reservation numbers that are greater than the number of available seats on the airplane. (Takagi[1], Sato, Sawaki[2], Cooper, Homem-de-Mello[13], Hayes, Miller[14], Talluri, Ryzin[15])

Boer et al.[5] stated that airline seat inventory control involves selling the right seats to the right people at the right time, and they categorized revenue management models as leg-based or network-based models. In addition, they analyzed network-based mathematical programming models and identified the need to include reservation limits and price-

* Waseda University, Tokyo, Japan

resetting methods in the stochastic programming model as the best approach to deal with uncertainties within the framework of revenue management.

Walczak et al.[6] demonstrated that overbooking can balance losses resulting from vacancy and boarding refusals; overbooking is determined by the expected income, probability that demand will exceed capacity, and the expected number of boarding refusals. Moreover, overbooking gives airlines the benefit of not only reducing overall costs by improving operational efficiency but also providing additional seats for passengers.

In Japan, the "flex traveler system" has been introduced as a response to situations when passengers are denied boarding due to overbooking. In the unlikely event of a shortage of seats, the airline will invite passengers who can accommodate changes in flights at the airport on the day of the flight. The system requires the airline to reimburse passengers who accept to such changes.

In this study, we propose an optimization model that uses stochastic programming to maximize the profit of an airline company by securing seats appropriately and employing the concept of overbooking.

2 Problem Description

2.1 Itinerary and Fight Leg

Airlines classify tickets that have identical departures and destinations into multiple classes and sell them at different rates. In this study, a combination of take off and landing (flight section) is referred to as a flight leg. In addition, a combination of the departure and the destination that does not include the transit airport is referred to as the itinerary, and a combination of the itinerary and the fare class is referred to as the origin, destination, fare class (ODF).

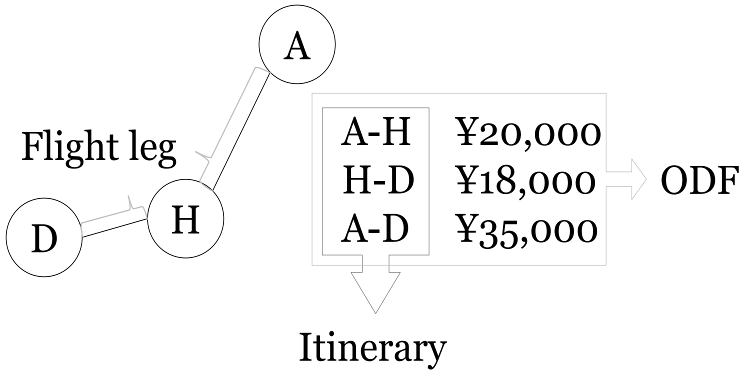

Figure 1: Example of itinerary and flight leg

2.2 Reservation

During the sale of tickets for various classes with identical departures and destinations, airlines adjust the number of reserved seats for each class. The number of reserved seats is the number of reservations accepted for each itinerary. If the actual demand is greater than the number of reserved seats, lost opportunities will occur, and if demand is less than the number of seats, losses due to vacancy will occur. Reservations tend to fill up beginning from the low-priced class; therefore, if the airline sells all the seats at the same price and the seats are sold out early, they will be unable to sell airline tickets to passengers willing to pay higher prices, which is a lost opportunity to boost revenue. In addition, because the fuel and maintenance costs required to operate the airplane do not change depending on the number of passengers, vacant seats at takeoff will result in losses; therefore, it is necessary to consider these losses as shown in Example 1.

Example 1

Fare ¥30,000 , Number of reserved seats 30 seats

・Demand 40 seats

$\rightarrow \underset{(Fare)}{¥30,000} \times \underset{(Insufficient)}{10seats} = ¥300,000$

Opportunity losses

・Demand 25 seats

$\rightarrow \underset{(Fare)}{¥30,000} \times \underset{(Vacant)}{5seats} = ¥150,000$

Vacant losses

2.3 Overbooking

Overbooking is permitted as a measure to minimize the number of vacant seats during takeoff caused by cancellations before boarding or no-shows. Overbooking is indicated by ensuring that the sum of the number of reserved seats in each class is greater than or equal to the number of airplane seats. Cancellation is allowed before boarding. Conversely, a no-show is a cancellation without permission, which refers to an instance wherein the passenger does not appear at the boarding gate by the scheduled time.

Airlines typically refund passengers who cancel their reservations with an amount equal to a part of the fare paid. In addition, if the airline company overbooks at the time of reservation and the actual number of cancellations is less than expected, boarding for a few passengers is refused. The company refunds the fare and reimburses passengers who have no alternative but to refuse boarding as shown in Example 2.

Example 2

Fare ¥10,000, Cancellation charge ¥8,000, Compensation ¥20,000

・for canceled passengers

$\rightarrow \underset{(Fare)}{¥10,000} - \underset{(Cancellationcharge)}{¥8,000} = \underset{(Refund)}{¥2,000}$

・for customers who have been denied boading

$\rightarrow \underset{(Fare)}{¥10,000} + \underset{(Compensation)}{¥20,000} = \underset{(RefundandCompensation)}{¥30,000}$

Allow 10 seats overbooking

・5 seats canceled

$\rightarrow \underset{(Lossoncancellationperperson)}{¥2,000} \times \underset{(Numberofcancellations)}{5seats} = \underset{(Lossoncancellation)}{¥10,000}$

$\underset{(Compensationforboadingdenialsperperson)}{¥30,000} \times \underset{(Numberofboadingrefusals)}{5seats} = \underset{(Lossoncancellation)}{¥150,000}$

Total ¥160,000.

3 Previous Study

3.1 Littlewood Model

In the Littlewood[3] model, the company sells tickets in two phases. In the first half of the sales, the tickets are sold at a discounted fare to price-sensitive, discount-seeking passengers (leisure travelers). Since the purpose of such passengers is to travel for leisure, there is a possibility of relatively early planning. However, passengers who buy tickets at a discounted price cannot change the reservation details; additionally, restrictions such as a high cancellation fare are levied. In the latter half of the sales, the reservation timing is often set prior to boarding, and the tickets are sold at regular fares for business passengers who dislike restrictions such as inability to change reservation details.

We assume that the demand of business passengers D is uncertain and that the flight can be fully booked by providing discounted fares. In addition, we assume that discount-seeking passengers make reservations before business passengers. If the airline accepts reservations on a first-come, first-serve basis until all seats are sold out, the discount-seeking passengers will occupy all the seats, and the airline's income will be low. To prevent this, Littlewood adopted the idea of a protection level y, which represents the number of seats reserved for high-paying customers. Seats apart from those reserved for the protection level y will be sold at a discounted fare. The remaining number of seats is referred to as the booking limit. Littlewood's formula is given in equation (1), where $F(\cdot)$, r_1, and r_2 denote the distribution function of D, fare for leisure traveler, and fare for business traveler, respectively.

$$F(y) = 1 - \frac{r_2}{r_1} \tag{1}$$

However, the Littlewood model is limited to two classes; additionally, even if the demand for high-priced classes is below the protection level, airlines do not sell tickets at discounted fares. Therefore, there is a possibility of take off with vacant seats.

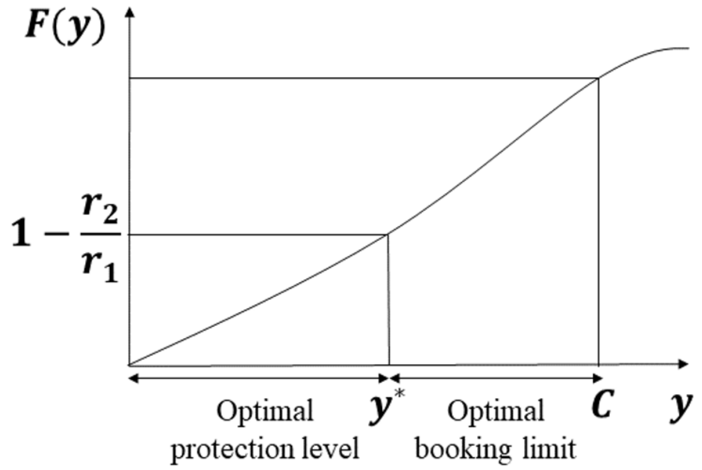

Figure 2: Optimal protection level and booking limit

3.2 Williamson Model

Williamson[4] used network-based models to optimize reservation management for the entire network using either a probabilistic mathematical programming problem (PMP) or a deterministic mathematical programming problem (DMP), both of which offer significant benefits. Since PMP uses stochastic demand, it needs to be solved using stochastic programming. Conversely, DMP simplifies the problem by replacing the uncertain demand with its expected value.

Parameter	
ODF	subscript for itinerary
l	subscript for flight leg
N	set of flight legs on the network
S_l	set of ODF available in the flight leg l
x_{ODF}	number of reserved seats in the ODF
C_l	number of seats of the flight leg l
D_{ODF}	probabilistic aggregated demand for ODF
f_{ODF}	fare of the ODF

The stochastic programming problem is formulated below. The term $\min\{x_{ODF}, D_{ODF}\}$ in equation (2) indicates that the objective function represents the number of reservations at takeoff. In PMP, the product of the fare and the actual number of reservations is assumed as the total revenue, and the expected value is maximized. In addition, inequality (3) is a capacity constraint to ensure that the total number of reserved seats is less than or equal to the number of airplane seats. The problem (PMP) can be expressed as

(PMP)

max

$$\mathrm{E}\sum_{ODF} f_{ODF} \min\{x_{ODF}, D_{ODF}\} \tag{2}$$

s.t.

$$\sum_{ODF\in S_l} x_{ODF} \leq C_l, \forall l \in N \tag{3}$$

$$x_{ODF} \geq 0, x_{ODF} \in \mathbb{Z}, \forall ODF \in S_l \tag{4}$$

Let D_{ODF} assume to take only the value of $d_{ODF,1} < d_{ODF,2} < \cdots < d_{ODF,K_{ODF}}$. The LP relaxation model, the stochastic linear programming (SLP) of the PMP, is given below. In equation (5), which represents the objective function of SLP, the product of the fare and the number of reserved seats is assumed as the profit obtained when all the seats are booked. Furthermore, the difference between the first term and the product of the fare and the probability that the demand is less than $d_{ODF,j}$ is assumed as the total revenue, and this difference is maximized. Inequality (6) is a capacity constraint to ensure that the total number of reserved seats is less than or equal to the number of airplane seats.

(SLP)

max

$$\sum_{ODF} f_{ODF} x_{ODF} - \sum_{ODF} f_{ODF} \sum_{j=1}^{K_{ODF}} P(D_{ODF} < d_{ODF,j}) \tag{5}$$

s.t.

$$\sum_{ODF\in S_l} x_{ODF} \leq C_l, \forall l \in N \tag{6}$$

$$x_{ODF} = \sum_{j=1}^{K_{ODF}} x_{ODF,j} \tag{7}$$

$$x_{ODF,1} \leq d_{ODF,j} \tag{8}$$

$$x_{ODF,j} \leq d_{ODF,j} - d_{ODF,j-1}, j = 2, .., K_{ODF} \tag{9}$$

$$x_{ODF,j} \geq 0, j = 1, ..., K_{ODF} \tag{10}$$

We can formulate a deterministic mathematical programming problem in a simplified manner. Equation (11), which shows the objective function of the DMP, considers the product of the fare and the number of reserved seats as the total profit and maximizes it. In addition, inequality (12) is a capacity constraint to ensure that the total number of reserved seats is less than or equal to the number of airplane seats. Inequality (13) indicates that the demand is constrained and that the number of reserved seats is less than or equal to the expected value of demand.

(DMP)

max

$$\sum_{ODF} f_{ODF} x_{ODF} \tag{11}$$

s.t.

$$\sum_{ODF \in S_l} x_{ODF} \leq C_l, \forall l \in N,, \tag{12}$$

$$x_{ODF} \leq \mathrm{E}D_{ODF}, \forall ODF \in S_l \tag{13}$$

$$x_{ODF} \geq 0, x_{ODF} \in \mathbb{Z}, \forall ODF \in S_l \tag{14}$$

However, these two models maximize profits at the time of booking and overlook cancellations subsequent to booking. Therefore, the disadvantage of these models is that they are ineffective in conditions of uncertainty in the number of future reservations.

4 Stochastic Programming Problem

Mathematical programming has been applied to many problems in various fields. However, for many actual problems, the data contain uncertainty and are thus represented as random variables because they represent information about the future. Decision-making under conditions of uncertainty involves potential risk. Stochastic programming deals with optimization under uncertainty. A stochastic programming problem with recourse is referred to as a twostage stochastic problem. In the first stage, a decision has to be made without complete information on random factors. After the value of random variables are known, recourse action can be taken in the second stage. We form the basic two-stage stochastic linear programming problem with recourse (SPR) as follows.

(SPR)

min

$$c^\top x + \mathscr{Q}(x)$$

s.t.

$$Ax = b, x \geq 0$$

$$\mathscr{Q}(x) = E_{\tilde{\xi}}[Q(x, \tilde{\xi})]$$

$$Q(x, \xi) = \min\{q(\xi)^\top y(\xi) \mid Wy(\xi) = h(\xi) - T(\xi)x, y(\xi) \geq 0\}, \xi \in \Xi$$

In the formulation of (SPR), c is a known n_1-vector, b a known m_1-vector, $q(>0)$ a known n_2-vector, and A and W are known matrices of size $m_1 \times n_1$ and $m_2 \times n_2$, respectively. The first stage decisions are represented by the n_1-vector x. We assume the m_2-random vector $\tilde{\xi}$ is defined on a known probablity space. Let Ξ be the support of $\tilde{\xi}$. Given a first stage decisions x, the realization of random vectors ξ of $\tilde{\xi}$ is observed. The second stage data ξ become known. Then, the second stage decision $y(\xi)$ must be taken to satisfy the constraints $Wy(\xi) \geq \xi - Tx$ and $y(\xi) \geq 0$. The second stage decision $y(\xi)$ is assumed to cause a penalty of q. The objective function contains a deterministic term $c^\top x$ and the expectation of the second stage objective. The symbol $E_{\tilde{\xi}}$ represents the mathematical expectation with respect to $\tilde{\xi}$, and the function $Q(x, \xi)$ is referred to as the recourse function in stage ξ. The value of the recourse function is given by solving a second stage linear programming problem. (Shiina[8])

5 Formulation of the New Model

Based on a study of the aforementioned models, we introduce factors such as lost opportunities and vacant seat loss at the time of reservation, cancellation loss before boarding, and boarding refusal loss, and we propose a stochastic programming model (Madansky[7], Möller, Römisch, Weber[9], Heitsch, Römisch[10], Higle, Sen[11], Chen, Homem-de-Mello[12]) that considers these. The objective function of this model can be calculated by minimizing losses from the product of the fares and the number of reserved seats, as well as optimizing the number of reserved seats to maximize the expected value of the total revenue of the airline, including overbooking in anticipation of cancellation.

Sets	
OD	set of itineraries (departure point arrival points)
F	set of classes (fares)
L	set of flight legs
Parameter	
C_{lj}	number of airplane seats on flight leg l, class j
f_{ij}	fares in itinerary i, class j
p_{ij}	loss on cancellation in itinerary i, class j(fare-cancellation fee)
q_{ij}	compensation for boarding denials in itinerary i, class j
Random variable	
$\tilde{\xi}_{ij}$	demand at the time of booking accordingto a normal distribution in the itinerary i, class j. Let Ξ_{ij} be the set of realization of values ξ_{ij}
$\tilde{\zeta}_{ij}$	Number of cancellations after booking according to Poisson distribution in itinerary i, class j. Let Z_{ij} be the set of realization of values ζ_{ij}
Variable	
x_{ij}	number of reserved seats in itinerary i, class j
$y^+_{ij}(\xi_{ij})$	number of insufficient seats at the time of booking in itinerary i, class j
$y^-_{ij}(\xi_{ij})$	number of surplus seats at the time of booking in itinerary i, class j
$w_{ij}(\xi_{ij},\zeta_{ij})$	number of boarding refusals in itinerary i, class j

Formulation

max

$$\sum_{i\in OD(l)}\sum_{j\in F} f_{ij}x_{ij} - \mathrm{E}_{\tilde{\xi}}[\sum_{i\in OD(l)}\sum_{j\in F} f_{ij}y^{+}_{ij}(\tilde{\xi}_{ij})] - \mathrm{E}_{\tilde{\xi}}[\sum_{i\in OD(l)}\sum_{j\in F} f_{ij}y^{-}_{ij}(\tilde{\xi}_{ij})]$$
$$-\mathrm{E}_{\tilde{\zeta}}[\sum_{i\in OD(l)}\sum_{j\in F} p_{ij}\tilde{\zeta}_{ij}] - \mathrm{E}_{\tilde{\xi}}\mathrm{E}_{\tilde{\zeta}}[\sum_{i\in OD(l)}\sum_{j\in F} q_{ij}w_{ij}(\tilde{\xi}_{ij},\tilde{\zeta}_{ij})] \tag{15}$$

s.t.

$$x_{ij} + y^{+}_{ij}(\xi_{ij}) - y^{-}_{ij}(\xi_{ij}) = \xi_{ij}, \forall i \in OD, \forall j \in F, \xi_{ij} \in \Xi_{ij} \tag{16}$$
$$\sum_{i\in OD(l)} (x_{ij} - y^{-}_{ij}(\xi_{ij}) - \zeta_{ij}) \le C_{lj} + \sum_{i\in OD(l)} w_{ij}(\xi_{ij},\zeta_{ij})$$
$$\forall l \in L, \xi_{ij} \in \Xi_{ij}, \zeta_{ij} \in Z_{ij} \tag{17}$$
$$x_{ij} \ge 0, x_{ij} \in \mathbb{Z}, \forall i \in OD, \forall j \in F \tag{18}$$
$$y^{+}_{ij}(\xi_{ij}), y^{-}_{ij}(\xi_{ij}) \ge 0, y^{+}_{ij}(\xi_{ij}), y^{-}_{ij}(\xi_{ij}) \in \mathbb{Z}, \forall i \in OD, \forall j \in F, \xi_{ij} \in \Xi_{ij} \tag{19}$$
$$w_{ij}(\xi_{ij},\zeta_{ij}) \ge 0, w_{i,j}(\xi_{ij},\zeta_{ij}) \in \mathbb{Z}, \forall i \in OD, \forall j \in F, \xi_{ij} \in \Xi_{ij}, \zeta_{ij} \in Z_{ij} \tag{20}$$

Equation (15) is the objective function of the proposed model. We consider the revenue from the number of reserved seats minus the opportunity, vacant seat, cancellation, and boarding refusal losses presented in equation (15) as the total revenue and maximize it. Equation (16) is the constraint of the demand in each itinerary and indicates that the sum of the number of reserved seats and the number of insufficient seats minus the number of surplus seats is equal to the demand. In addition, the number of surplus seats and the number of cancellations subtracted from the number of reserved seats in inequality (17) represents the number of passengers at the time of boarding. Inequality (17) is a capacity constraint in each flight leg and indicates that the number of passengers at the time of boarding is less than the combined sum of the airplane capacity and number of boarding refusals.

Setting random variables

We assume that the demand $\tilde{\xi}_{ij}$ follows a normal distribution $N(\mu_{ij}, \sigma_j^2)$ and is represented by the following probability density function:

$$f(\xi_{ij}) = \frac{1}{\sqrt{2\pi\sigma_j^2}}\exp[-\frac{(\xi_{ij}-\mu_{ij})^2}{2\sigma_j^2}]$$

The expected value of this probability density function is given by $E(\xi_{ij}) = \mu_{ij}$. The number of cancellations $\tilde{\zeta}_{ij}$ follows the Poisson distribution $Po(\lambda_{ij})$ and is represented by the following probability:

$$P(\zeta_{ij}) = \frac{e^{\lambda_{ij}}\lambda_{ij}^{\zeta_{ij}}}{\zeta_{ij}!}$$
$$\lambda_{ij} = c_j x_{ij}$$

The symbol c_j represents the probability of cancellation per customer. The expected value of the distribution function is given by $E(\zeta_{ij}) = \lambda_{ij}$. For the probability distribution used in this study, the upper and lower limits ($a \le x \le b$) were set, and the truncated distribution expressed by the equations below were applied. Function $g(x)$ and function $F(x)$

indicate the density and cumulative distribution functions of the random variables, respectively.
Density function:

$$\frac{g(x)}{F(b)-F(a)}$$

Cumulative distribution function:

$$\frac{\int_a^x g(t)dt}{F(b)-F(a)} = \frac{F(x)-F(a)}{F(b)-F(a)}$$

Expected value:

$$\frac{\int_a^b xg(x)dx}{F(b)-F(a)}$$

6 Numerical Experiments

The following two models were compared with the proposed model that uses the stochastic programming problem:

6.1 Littlewood Model Considering Cancellation

The optimal number of reserved seats was calculated using the Littlewood's concept[3] of optimal protection level. The original Littlewood model can be extended by introducing cancellations.

Variable

x'_{ij}	number of reserved seats in itinerary i, class j.
$y'^{+}_{ij}(\xi_{ij})$	number of insufficient seats at the time of booking in itinerary i, class j.
$y'^{-}_{ij}(\xi_{ij})$	number of surplus seats at the time of booking in itinerary i, class j.

Considering the formulas based on Littlewood's formula (1),

$$F(x'_{i1}) = 1 - \frac{f_{i2}}{f_{i1}} \tag{21}$$

and

$$F(x'_{i1}) = \Phi(\frac{x'_{i1}-\mu_{i1}}{\sigma}), \tag{22}$$

we obtain

$$1 - \frac{f_{i2}}{f_{i1}} = F(x'_{i1}) = \Phi(\frac{x'_{i1}-\mu_{i1}}{\sigma}).$$

Furthermore, using z_i (which is defined as a value such that $\Phi(z_i) = (1 - \frac{f_{i2}}{f_{i1}})$), we determine the decision variables as follows:

$$x'_{i1} = \mu_{i1} + z_i\sigma$$

$$x'_{i2} = \mu_{i1} + \mu_{i2} - x'_{i1}$$

$$y'^{+}_{ij}(\xi_{ij}) = \max(0, \mu_{ij} - x'_{ij})$$

$$y'^{-}_{ij}(\xi_{ij}) = \max(0, x'_{ij} - \mu_{ij})$$

We substitute these into the formula for the objective function to obtain the maximum profit.

$$\sum_{i\in OD(l)}\sum_{j\in F} f_{ij}x'_{ij} - \sum_{i\in OD(l)}\sum_{j\in F} f_{ij}y'^{+}_{ij}(\bar{\xi}_{ij}) - \sum_{i\in OD(l)}\sum_{j\in F} f_{ij}y'^{-}_{ij}(\bar{\xi}_{ij}) - \sum_{i\in OD(l)}\sum_{j\in F} p_{ij}\zeta_{ij} \quad (23)$$

6.2 Deterministic Model

In the deterministic model, the profit is maximized by the optimum number of reserved seats x^d obtained using the expected value of the normal distribution for the demand and the expected value of the Poisson distribution for the number of cancellations without considering the fluctuation.

Equation (15) is an objective function of the original problem. We consider the revenue from the number of reserved seats minus the opportunity, vacant seat, cancellation, and boarding refusal losses presented in equation (24) as the total revenue and maximize it. Equality (25) is the constraint of the demand in each itinerary and indicates that the sum of the number of reserved seats and the number of insufficient seats minus the number of surplus seats is equal to the demand. In addition, the number of surplus seats and the number of cancellations subtracted from the number of reserved seats in inequality (26) represents the number of customers at the time of boarding. Inequality (26) is a capacity constraint in each flight leg and indicates that the number of customers at the time of boarding is less than the sum of the airplane capacity and the number of boarding refusals.

Parameters

x^d_{ij}	number of reserved seats at the time of booking in itinerary i, class j
$y^{d+}_{ij}(\xi_{ij})$	number of insufficient seats at the time of booking in itinerary i, class j
$y^{d-}_{ij}(\xi_{ij})$	number of surplus seats at the time ofbooking in itinerary i, class j

Formulation

$$\max \sum_{i\in OD(l)}\sum_{j\in F} f_{ij}x^d_{ij} - \sum_{i\in OD(l)}\sum_{j\in F} f_{ij}y^{d+}_{ij} - \sum_{i\in OD(l)}\sum_{j\in F} f_{ij}y^{d-}_{ij} - \sum_{i\in OD(l)}\sum_{j\in F} p_{ij}\mathrm{E}(\zeta_{ij}) - \sum_{i\in OD(l)}\sum_{j\in F} q_{ij}w^d_{ij} \quad (24)$$

s.t.

$$x^d_{ij} + y^{d+}_{ij} - y^{d-}_{ij} = \mathrm{E}(\xi_{ij}),\ \forall i\in OD, \forall j\in F, \xi_{ij}\in \Xi_{ij} \quad (25)$$

$$\sum_{i\in OD(l)} (x^d_{ij} - y^{d-}_{ij} - \mathrm{E}(\zeta_{ij})) \le C_{lj} + \sum_{i\in OD(l)} w^d_{ij}, \forall l\in L, \xi_{ij}\in \Xi_{ij}, \zeta_{ij}\in Z_{ij} \quad (26)$$

$$x^d_{ij} \ge 0, x^d_{ij}\in \mathbb{Z}, \forall i\in OD, \forall j\in F \quad (27)$$

$$y^{d+}_{ij}, y^{d-}_{ij} \ge 0, y^{d+}_{ij}, y^{d-}_{ij} \in \mathbb{Z}, \forall i\in OD, \forall j\in F, \xi_{ij}\in \Xi_{ij} \quad (28)$$

$$w^d_{ij} \ge 0, w^d_{ij}\in \mathbb{Z}, \forall i\in OD, \forall j\in F, \xi_{ij}\in \Xi_{ij}, \zeta_{ij}\in Z_{ij}. \quad (29)$$

6.3 Data Setting

In this study, the network shown in figure (3) was used. We set seven itineraries by connecting four flight legs and connecting five airports, A, B, C, D, and H. Each itinerary includes

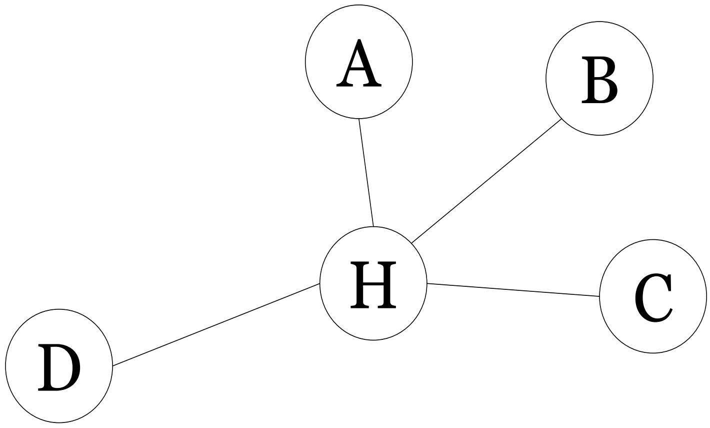

Figure 3: Exampl of network

one or two flight legs. To travel from airports A, B, and C to airport D, there is no direct flight; therefore, it is necessary to go through airport H. We limited our study to two classes of flights. The number of seats in each flight leg is listed in table 1. In addition, the expected value of the fare and demand for each itinerary are listed in table 2. The cancellation loss

Table 1: Number of seats on each flight leg

Leg number	Leg	Number of seats Class1	Class2
1	A-H	20	145
2	B-H	15	80
3	C-H	20	145
4	H-D	94	263

Table 2: Fare and expected value of the demand on each flight leg

OD Number	OD	Fare (yen) Class1	Fare (yen) Class2	Expected value Class1	Expected value Class2
1	A-H	18590	12790	18	131
2	B-H	16590	12490	14	72
3	C-H	20190	14690	18	131
4	H-D	38310	9510	72	385
5	A-H-D	50200	19900	2	15
6	B-H-D	31360	19600	2	8
7	C-H-D	49900	20000	2	15

and boarding refusal fee can be set as shown below. In addition, the fluctuation of demand the cancellation probability of each class are shown in the table 3. Loss of cancellation fee

・Class 1： (fare - 440yen) × 0.6

・Class 2： (fare50%) × 0.8

Loss of boarding refusal fee
・fare + 10,000yen

Table 3: Fluctuations in demand and cancellation probability for each class

class	σ_j^2	$c_j(\%)$
1	5	15
2	30	5

7 Result and Discussion

Table 4 lists the total profits of the three models; apparently the total profits obtained by the stochastic programming model are the maximum. Table 5 shows the number of seats of each ODF, as determined by each model. In the Littlewood model, the number of reserved seats is determined only by the difference in the fare between Class 1 and Class 2; therefore, it presents the disadvantage of securing an excessive number of Class 1 seats with high fares, resulting in a large loss of opportunities, vacant seat losses, and losses on cancellation.

Table 6 shows the revenue for each itinerary for each model. In OD four of Littlewood's model resulted in a loss of 959×10^3yen. If the fare difference between the two classes is large and the demand for itineraries is high, the profit may be negative.A negative profit on just one itinerary is considered to be very ineffective when the network is further expanded.

The stochastic model reserves seats effectively using the demands shown in table 7, which are determined considering fluctuations. Depending on the itinerary, more seats than what is available on the airplane are sold; however, it is possible to minimize total loss.

In addition, the number of cancellations differs depending on whether fluctuations are considered; therefore, it is apparent that the stochastic programming model is considerably more realistic compared with the deterministic model.

Finally, boarding refusals did not occur on any of the three models. The reason is that the loss to the total profit is large. Boarding refusals the realiability reliability of the airline and tends to be avoided as much as possible. Furthremore, the amount of compensation for boarding refusals varies, depending on the circumstance. Consequently, if the compensation is relatively low and it is possible to guide to the next flight, boarding refusal may occur.

Table 4: Total revenue for each model

Model	Total revenue(yen)
Littlewood	423,270
Deterministic	11,391,600
Stochastic	11,758,300

8 Summary and Future Plans

In this study, we proved that it is possible to maximize the expected value of total revenue by securing seats and using an optimization model based on a stochastic programming model. Furthermore, we compared the results obtained using the Littlewood model (which

Table 5: Number of reserved seats for each model

OD	Littlewood		Deterministic		Stochastic	
No.	Class1	Class2	Class1	Class2	Class1	Class2
1	65	84	18	131	17	145
2	48	38	14	72	12	59
3	68	81	18	131	17	145
4	398	0	89	227	114	289
5	8	8	2	14	1	6
6	1	8	1	8	1	3
7	8	8	2	14	1	6

Table 6: Profit for each model(10^3 yen)

OD	Littlewood		Deterministic		Stochastic	
No.	Class1	Class2	Class1	Class2	Class1	Class2
1	248	405	305	1,642	3,302	1,809
2	115	2	212	881	185	679
3	22	241	331	1,886	282	2,005
4	1,200	-2,159	2,106	2,116	3,542	2,548
5	74	31	91	273	50	119
6	31	145	29	154	0	59
7	46	24	91	274	50	120

Table 7: Demand for stochastic programming model

OD	Demand	
number	class1	class2
1	17	145
2	12	59
3	17	145
4	114	289
5	1	6
6	0	3
7	1	6

allocates seats based on the Littlewood formula) and the deterministic model (which does not consider fluctuations). The Littlewood model presents the disadvantage that the fare difference between the two classes strongly affects the number of seats reserved. Therefore, airlines will reserve an excessive number of high-priced class seats and increase lost opportunities and vacant seat losses. The stochastic programming model proposed in this study is a practical model that enables realistic predictions by examining multiple scenarios, unlike the deterministic model that does not consider fluctuations and examines only one scenario.

In the future, sales methods are expected to increase various aspects; moreover, it is expected that additional classes of airline tickets will be intoruduced. In addition, it is now possible to easily book airline tickets using the internet; hence, it is necessary to design a revenue management method capable of responding flexibly.

Future research areas include application to networks that consider larger itineraries, examination of scenarios other than the those considered in this study, namely cancellation and boarding refusal losses, and the application of considerably realistic introduction methods.

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