

holds between (X'_1, X'_5) and (X'_1, X'_7) . But, the dependent relation does not hold between (X'_1, X'_4) and (X'_1, X'_7) , in which only Boolean variable (b) is derived in Figure 14. The Boolean variable (a, b) on the degenerate convex cone in Figure 12 removes (X'_3, X'_1) and (X'_5, X'_1) . However, the variable (b) in (X'_4, X'_1) is remained.

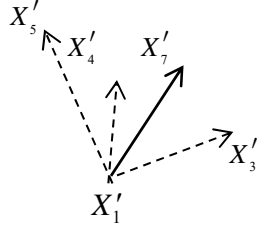


Figure 14: Dependent relations on degenerate convex cones

Thus, the variable (b) is selected by the hyperplane classification A_2 . Boolean sum terms $(b + c)$ and $(c + d)$ are derived based on the hyperplane A_1 and a term (b) on the hyperplane A_2 . By product terms, complete reducts become $\{bc, bd\}$.

Theorem 5.4 Dimensionality reduction of variables for reducts is realized by convex cones on the nearest neighbor relations, which are generated by the linear subspaces.

6 Classification of Reduced Variables in Threshold Network

The classification of instances is realized using linear classifiers with reduced variables, which are derived based on the geometrical reasoning in the previous section 5. The reduced variables for reducts are obtained based on the convex cones of the ne

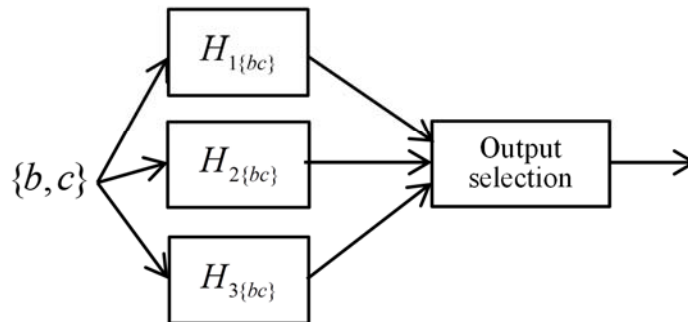


Figure 15: Hyperplanes generated on convex cones with nearest neighbor relations

est neighbor relations in Figure 9. Under the restricted conditions of inequalities with four variables of instances in Table 6, the relation of the hyperplane with the reduced variables are expected to be made clear. Introducing slack variables in the reduced ones of the inequalities, we can derive the hyperplanes with reduced variables. In the section 5, the reduction of variables are carried out, based on the geometric reasoning on convex cones of nearest neighbor relations. As the reduced variables, a reduct $\{bc\}$ of variables b and c is obtained

in the section 5.4. By using this reduct $\{bc\}$, a classification of the reduced variables is performed as shown in Figure 15. In Figure 15, three hyperplanes, $H_{1\{bc\}}, H_{2\{bc\}}, H_{3\{bc\}}$ are shown. The hyperplane A_1 in Figure 7 is shown in the equation (4), which is also in the degenerated convex in Figure 10. To compute the hyperplane $H_{1\{bc\}}$ with reduced two variables b and c , the hyperplane A_1 with four variables, a, b, c and d in the equation of the hyper plane A_1 is started,

$$A_1: \quad (3/2)a - 2b - 2d = 0 \quad (18)$$

, where $\varepsilon=1$ in the equation (7) is set. Since the reduced variables $\{b, c\}$ consists of two variables, three vertices x_3, x_6, x_7 on the convex in Figure 9 are taken on the convex in Figure 9 as the first reduced hyperplane, $H_{1\{bc\}}$. For the instance, x_3 , the following equation(19) holds using the hyperplane A_1 . Since $x_3 = (2, 2, 0, 0)$ belongs to -1 class,

$$(3/2) \cdot 2 - 2(1 + \Delta\xi_1) \cdot 2 + \Delta\xi_2 + \theta \leq 0 \quad (19)$$

, where $\Delta\xi_1$ and $\Delta\xi_2$ are slack variables. From the equation (19), we set

$$-4\Delta\xi_1 + \theta = -1 \quad (20)$$

Similarly, since $x_6 = (2, 1, 1, 0)$ belongs to +1 class,

$$(3/2) \cdot 2 - 2(1 + \Delta\xi_1) \cdot 1 + \Delta\xi_2 \cdot 1 - 2 \cdot 0 + \theta > 0 \quad (21)$$

Then, we set

$$-2\Delta\xi_1 + \Delta\xi_2 + \theta = +1 \quad (22)$$

Similarly, since $x_7 = (2, 1, 2, 1)$ belongs -1 class,

$$(3/2) \cdot 2 - 2(1 + \Delta\xi_1) \cdot 1 + \Delta\xi_2 \cdot 2 - 2 \cdot 1 + \theta \leq 0 \quad (23)$$

Then, we set

$$-2\Delta\xi_1 + 2\Delta\xi_2 + \theta = 0 \quad (24)$$

From equations (20), (22) and (24), $\Delta\xi_1 = (+3/2)$, $\Delta\xi_2 = -1$ and $\theta = +5$ are obtained.

Thus, from equations (19),(21) and (23), a hyperplane with reduced variables $\{b, c\}$ is given by coefficient products of each slack variables

$$-2 \cdot (+3/2)b - 1 \cdot c + 5 = -3b - c + 5 = 0 \quad (25)$$

The equation (25) of the hyperplane with reduced variables $\{b, c\}$, classifies to the respective classes of x_1, x_2, x_3, x_4, x_6 and x_7 , but it does not satisfy the class of x_5 . Then, we need another hyperplanes different from $H_{1\{bc\}}$. From another three vertices x_5, x_3, x_6 on the convex in Figure 9 as the second reduced hyperplane, $H_{2\{bc\}}$ are computed, which classifies to the respective classes of x_1, x_2, x_3, x_5 and x_6 , but it does not satisfy the classes of x_4 and x_7 . Similarly, from another three vertices x_5, x_3, x_7 on the convex in Figure 9 as the third reduced hyperplane, $H_{3\{bc\}}$ are computed, which classifies to the respective classes of x_3, x_4, x_5 and x_7 , but it does not satisfy the classes of x_1, x_2 and x_6 . All the correct classifications of the classes are carried out by majority voting using these three hyperplanes with reduced variables, $H_{1\{bc\}}, H_{2\{bc\}}, H_{3\{bc\}}$ in Figure 15. Then, all the instances are classified, correctly. Combining these three hyperplanes with reduced variables, another correct classification is also performed using the output selection by the comparison of their values.

7 Conclusion

In this paper, the reduction of data variables and the classification through the nearest neighbor relations are proposed in the threshold networks. For the threshold function, the method of reduction of variables is characterized by the Boolean operations of vectors of the nearest neighbor relations. For the data with general values in threshold networks, reduction of variables are realized based on convex cones made of the nearest neighbor relations in threshold networks. It is shown that the nearest neighbor relations derives approximated reducts of reduced variables. To derive complete reducts based on the approximated reducts, the degenerate convex cones are generated in the linear subspaces based on the nearest neighbor relations. Then, the dependent relations and the algebraic operations of edges on the degenerate convex cones are developed in the linear subspaces. The classification using the reduced variables is realized in the threshold networks.

References

- [1] Z. Pawlak, "Rough Sets," International Journal of Computer and Information Science, vol.11, 1982, pp.341-356.
- [2] Z. Pawlak and R. Slowinski, "Rough Set Approach to Multi-attribute Decision Analysis," European Journal of Operations Research 72, 1994, pp.443-459 .

- [3] A. Skowron and C. Rauszer, “The Discernibility Matrices and Functions in Information Systems,” in *Intelligent Decision Support- Handbook of Application and Advances of Rough Sets Theory*, pp.331-362, Kluwer Academic Publishers, Dordrecht, 1992
- [4] A. Skowron and L. Polkowski, “Decision Algorithms, A Survey of Rough Set Theoretic Methods,” *Fundamenta Informatica*, 30/3-4, pp. 1997, 345-358.
- [5] Ky Fan: *On Systems of Linear Inequalities, Linear Inequalities and Related Systems*, edited by H. W. Kuhn and A.W. Tucker, Princeton University Press, 99-156(1966)
- [6] T. M. Cover and P.E. Hart, “Nearest Neighbor Pattern Classification,” *IEEE Transactions on Information Theory*, Vol.13, No.1, 1967, pp.21-27.
- [7] F.P. Preparata and M.I. Shamos, *Computational Geometry*, Springer Verlag, 1993
- [8] W. Prenowitz and J.Jantosciak, *Join Geometries, A Theory of Convex Sets and Linear Geometry*, Springer Verlag, 2013,
- [9] N. Ishii, I. Torii, K. Iwata, K.Odagiri, T. Nakashima: *Generation and Nonlinear Mapping of Reducts-Nearest Neighbor Classification*. Chapter 5 in *Advances in Combining Intelligent Methods*, Springer Verlag, 93-108(2017)
- [10] N. Ishii, I.Torii, K. Iwata, K. Odagiri, T. Nakashima: *Generation of Reducts Based on Nearest Neighbor Relations and Boolean Reasoning*, HAIS2017, LNCS vol.10334, Springer, 391-401(2017)
- [11] N. Ishii, I. Torii, N. Mukai, K. Iwata and T. Nakashima, “Generation of Reducts and Threshold Function Using Discernibility and Indiscernibility Matrices”, *Proc. ACIS-SERA IEEE Comp. Soc.*, 55-61(2017)
- [12] A.V.Levitin, *Introduction to the Design and Analysis of Algorithms*, Addison Wesley, 2002
- [13] A. De, I. Diakonikolas, V. Feldman, R.A. Servedio, “Nearly Optimal Solutions for the Chow Parameters Problem and Low-weight Approximation of Halfspaces”, *J.ACM*, Vol.61,No.2, 2014, pp.11:1-11:36.
- [14] S.T.Hu, *Threshold Logic*, University of California Press,1965
- [15] N. Ishii, I. Torii, N. Mukai, K. Iwata and T. Nakashima, “Incremental Reducts Based on Nearest Neighbor Relations and Linear Classification”, *Proc. IIAI-SCAI IEEE Comp. Soc.*, 528-533(2019)